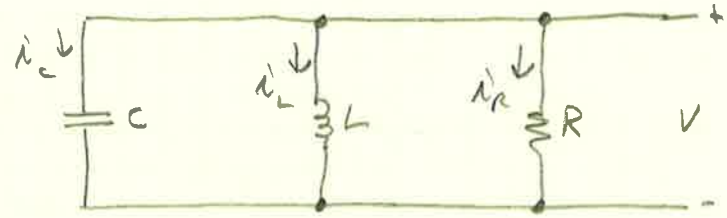
$L = 10mH$

for $t \ge 0$,

$v(t) = 40e^{-1000t} - 90e^{-4000t}$ V



a) Find ω_0, α, C, and R

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1000$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -4000$$

adding, $-5000 = -2\alpha \Rightarrow \boxed{\alpha = 2500 \text{ rad/s}}$

$$-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1000$$
$$-2500 + \sqrt{2500^2 - \omega_0^2} = -1000 \Rightarrow \boxed{\omega_0 = 2000 \text{ rad/s}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C = 25\mu F}$$

$$\alpha = \frac{1}{2RC} \Rightarrow \boxed{R = 8\Omega}$$

b) FIND $i_R(t)$, $i_L(t)$, and $i_C(t)$ for $t \ge 0^+$

$$i_R(t) = \frac{v(t)}{R} = 5e^{-1000t} - 11.25e^{-4000t} \text{ A}$$

$$i_C(t) = C\frac{dv_c}{dt} = (25\times10^{-6})\left[-40000e^{-1000t} + 90(4000)e^{-4000t}\right]$$
$$= -e^{-1000t} + 9e^{-4000t} \text{ A}$$

$$i_L(t) = -(i_r + i_c) = -4e^{-1000t} + 2.25e^{-4000t} \text{ A}$$

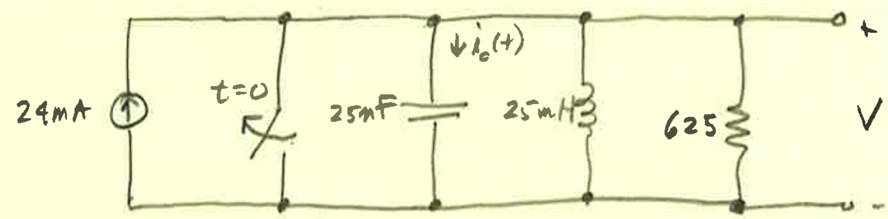
8.25 | Nillson 11th

For $t \geq 0$, Find $v(t)$ + $i_c(t)$

$v_c(0) = i_L(0) = 0$

$i_c(\infty) = V(\infty) = 0$

$i_L(\infty) = 24mA$



$$\alpha = \frac{1}{2RC} = \frac{1}{2(625)(25\times10^{-9})} = 32{,}000 \text{ Rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25\times10^{-3}(25\times10^{-9})}} = 40{,}000 \text{ Rad/sec}$$

$\omega_0 > \alpha$ So roots are underdamped $\quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 24{,}000 \text{ Rad/s}$

$$V_c(t) = v_{cf} + e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$= 0 + e^{-32{,}000t}(B_1 \cos 24{,}000t + B_2 \sin 24{,}000t)$$

$V_c(0) = 0 = B_1$

$V_c(t) = e^{-32{,}000t} B_2 \sin 24{,}000t$

$i_C(0^+) = 24mA$ since i_L can't change from 0 @ 0^- and i_r " " " " " " or V_c change instantaneously

$$i_c(0^+) = C\frac{dV_c}{dt}\bigg|_{t=0^+} = C\left[e^{-32{,}000t} B_2 (24000) \cos 24000t + B_2 \sin \cdots\right]$$

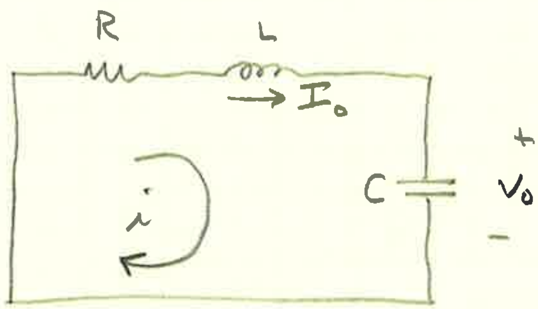
$24mA = 25\times10^{-9}(24000) B_2 \Rightarrow B_2 = 40$

$$\boxed{V(t) = 40e^{-32{,}000t} \sin 24{,}000t \ V}$$

$$i_c(t) = C\frac{dV_c(t)}{dt} = C\left[40(24{,}000)e^{-32000t} \cos 24{,}000t + \sin 24{,}000t\,(40)(-32000)e^{-32000t}\right]$$

$$\boxed{i_c(t) = 24e^{-32{,}000t} \cos 24{,}000t - 32e^{-32{,}000t} \sin 24{,}000t \ mA}$$

$i = B_1 e^{-2000t} \cos 1500t + B_2 e^{-2000t} \sin 1500t$



$C = 80 nF$
$I(0) = 7.5 mA$
$V_c(0) = -30V$

Find $R, L, B_1, + B_2$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 1500 \qquad \alpha = 2000$

$\omega_0^2 = \omega_d^2 + \alpha^2 \Rightarrow \omega_0 = 2500$ rad/sec

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \boxed{2H = L}$

$\alpha = \frac{R}{2L} \Rightarrow R = 2L\alpha = 2(2)(2000) = \boxed{8000\Omega = R}$

$i(0^-) = i(0^+) = 7.5mA$

$i(0^+) = \boxed{7.5mA = B_1}$

$i = 0.0075 e^{-2000t} \cos 1500t + B_2 e^{-2000t} \sin 1500t$

@ time 0, $V_r + V_L + V_c = 0 \Rightarrow R(i_0) + V_L - 30 = 0$
$-8000(.0075) + 30 = V_L(0)$
$V_L(0) = -30V$

$V_L = L\frac{di_L}{dt}$

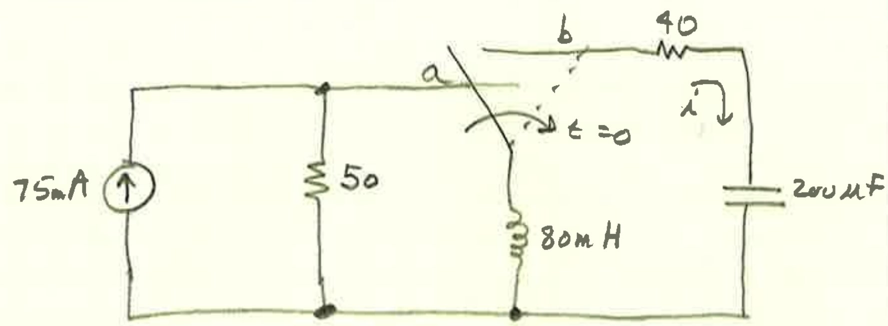
$-30 = V_L(0^+) = 2\Big[.0075 e^{-2000t}(-\sin 1500t)(1500) + \cos 1500t(-2000)(.0075)e^{-2000t} +$
$B_2 e^{-2000t} \cos 1500t\,(1500) + \sin 1500t\,(-2000 B_2 e^{-2000t})\Big]\Big|_{t=0}$

$-30 = 2[-2000(.0075) + B_2(1500)]$

$\boxed{B_2 = 0}$

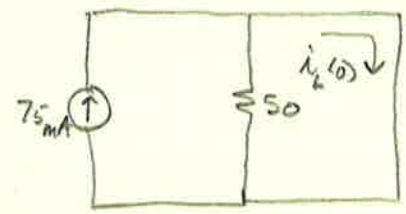
$\boxed{i(t) = .0075 e^{-2000t} \cos 1500t \ A}$

switch has been at position a for a long time at $t=0$, switch moves to position b. Find $i(t)$ for $t \geq 0$.



$t<0$: $i_L(0) = 75\,mA$
$V_c(0) = 0\,V$

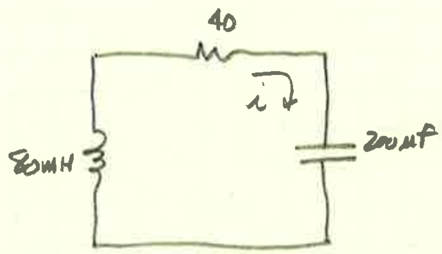
$i(0) = -75\,mA$



$t>0$:

$\alpha = \dfrac{R}{2L} = 250$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 250$



critically damped

$$i(t) = A_1 t e^{-250t} + A_2 e^{-250t}$$

$@t=0,\ i = -75\,mA = A_2$

$$i(t) = A_1 t e^{-250t} - 0.075 e^{-250t}$$

KVL $@t=0$: $V_L + V_R + V_C = 0$

$$L\frac{di_L}{dt} + (-.075)(40) + 0 = 0$$

$$L\left[A_1 t(-250 e^{-250t}) + A_1 e^{-250t} - .075(-250)e^{-250t}\right]_{t=0} = 3$$

$$.08\left[A_1 - .075(-250)\right] = 3$$

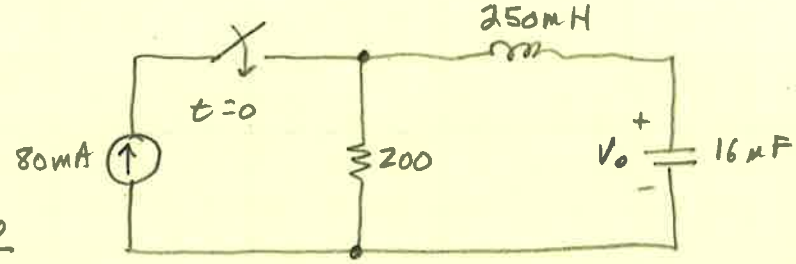
$$A_1 = 18.75$$

$$\boxed{\therefore\ i(t) = 18.75 t e^{-250t} - .075 e^{-250t}\ A}$$

8.50 Nilsson 11^{th}

initial energy = 0

find $V_0(t)$ for $t \geq 0$



$$\alpha = \frac{R}{2L} = \frac{200}{2(.25)} = 400 \ \frac{rad}{s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{.25(16 \times 10^{-6})}} = 500 \ \frac{rad}{s}$$

$\omega_0 > \alpha$ so underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 300 \ rad/s$$

$$v_0 = V_f + B_1 e^{-400t} \cos 300t + B_2 e^{-400t} \sin 300t$$

$$v_0(\infty) = 200(.08) = 16V$$
$$v_0(0) = 16 + B_1 = 0 \quad \Rightarrow B_1 = -16$$

$$\therefore \quad v_0(t) = 16 - 16e^{-400t} \cos 300t + B_2 e^{-400t} \sin 300t$$

we know the current through the capacitor = 0 @ t = 0

$$i_c = c \frac{dv_c}{dt}\Big|_{t=0} = 0 = C\left(-16e^{-400t}\left(300(-\sin 300t)\right) + \cos 300t(-16)(-400)e^{-400t}\right)$$
$$+ c\left(B_2 e^{-400t} \cos 300t\,(300) + \sin 300t(-400B_2 e^{-400t})\right)$$

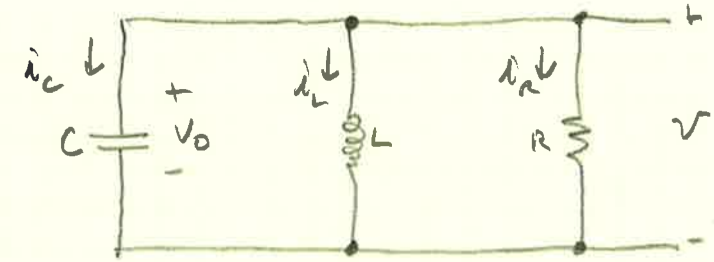
evaluate @ t = 0 : $\quad 0 = (-16)(-400) + 300B_2$

$$B_2 = -21.33$$

$$\boxed{V_0(t) = 16 - 16e^{-400t} \cos 300t - 21.33e^{-400t} \sin 300t \ \ V}$$

$R = 125\,\Omega$
$L = 200\,mH$
$C = 5\,\mu F$

$i_L(0) = -0.3A$
$V_c(0) = 25V$



a) Find the initial current in each element.

$\boxed{i_L = -300mA}$

$i_R = \frac{V(0)}{125} = \frac{25}{125} = \boxed{200mA = i_r}$

$i_L + i_c + i_r = 0 \Rightarrow i_c = -(i_L + i_R) = \boxed{100mA = i_c}$

b) Find $V(t)$ for $t \geq 0$

$\alpha = \frac{1}{2RC} = 800 \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 1000$

$\omega_0 > \alpha$ so system is underdamped

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 600$

$v_c(t) = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t$

Finding $B_1 + B_2$:

$v_c(0) = 25 = B_1$

Node equation @ $t=0$: $i_c + i_L + i_R = 0$

$C\frac{dv}{dt}\Big|_{t=0} + (-300mA) + \frac{25}{125} = 0$

$(5 \times 10^{-6})\left[-800B_1 + B_2\,600\right] = 0.1$

$B_2 = 66.67$

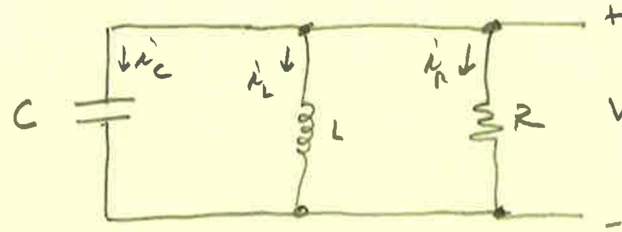
$$\boxed{v = 25e^{-800t} \cos 600t + 66.67e^{-800t} \sin 600t \ V}$$

c) Find $i_L(t)$ for $t \geq 0$

$i_L = \frac{1}{L}\int_0^t v(t)\,dt = \boxed{-300e^{-800t} \cos 600t - 191.7e^{-800t} \sin 600t \ mA}$

the voltage response is

$$v(t) = D_1 t e^{-80t} + D_2 e^{-80t} \quad t \geq 0$$



$I_L(0) = -25\,mA$
$V_c(0) = 5V$
$R = 50\Omega$

a) Find C, L, D_1, + D_2

Circuit is critically damped $\Rightarrow s_1 = s_2 = -\frac{1}{2RC} = -80$

$$C = \frac{1}{2(80)(R)} = \boxed{125\ \mu F}$$

For critically damped, $\frac{1}{(2RC)^2} = \frac{1}{LC}$ or $L = 2^2R^2C = \boxed{1.25\,H}$

$V(0) = \boxed{5 = D_2}$ So

$$v(t) = D_1 t e^{-80t} + 5e^{-80t}$$

@ $t=0$, do node equation.

$$i_c(0) + i_L(0) + i_r(0) = 0$$

$$C\frac{dV_c(t)}{dt} + (-25mA) + \frac{V(0)}{R} = 0$$

$$125 \times 10^{-6}\left[D_1 t(-80)e^{-80t} + e^{-80t}D_1 + 5(-80)e^{-80t}\right]\Big|_{t=0} - 25mA + \frac{5}{50} = 0$$

$$125 \times 10^{-6}\left[D_1 - 400\right] - 25mA + 0.1 = 0$$

$$\boxed{D_1 = -200}$$

b) find $i_c(t)$ for $t \geq 0^+$

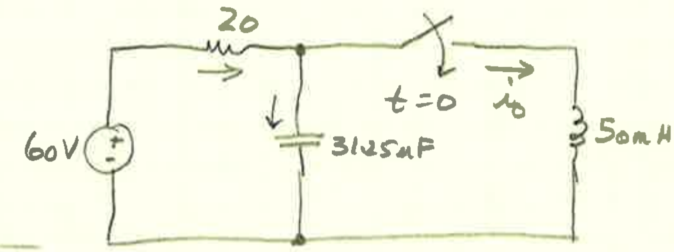
$$i_c = C\frac{dV}{dt} = C\left[-200t(-80)e^{-80t} + e^{-80t}(-200) + 5(-80)e^{-80t}\right]$$

$$= C\left[16000te^{-80t} - 600e^{-80t}\right]$$

$$\boxed{i_c = \left(2te^{-80t} - .075e^{-80t}\right)A}$$

Switch closes at $t=0$

Find $i_o(t)$ for $t \geq 0$



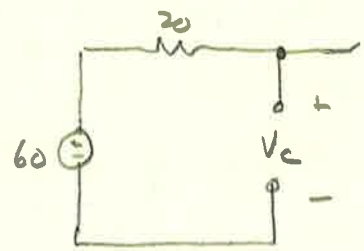
$V_c(0) = 60V$

$i_L(0) = 0A$

$i_F(\infty) = \frac{60}{20} = 3A$

$\alpha = \frac{1}{2RC} = 800$

$\omega_o = \sqrt{\frac{1}{LC}} = 800$



So critically damped

$i_o(t) = i_F + A_1 t e^{-800t} + A_2 e^{-800t}$

$i_o(0) = 0 = 3 + A_2 \Rightarrow A_2 = -3$

$\therefore i_o(t) = 3 + A_1 t e^{-800t} - 3e^{-800t}$

$\left.\frac{di_o(t)}{dt}\right|_0 = A_1 t(-800 e^{-800t}) + A_1 e^{-800t} - 3(-800)e^{-800t} = \left.\frac{V_L}{L}\right|_{t=0}$

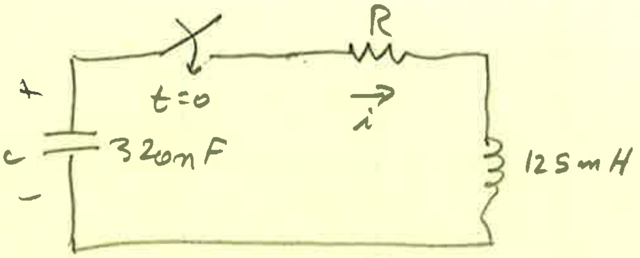
$A_1 + 2400 = \frac{60}{.05} \Rightarrow A_1 = -1200$

$$\boxed{i_o(t) = 3 - (1200t + 3)e^{-800t} \text{ A}}$$

	8.42	Nilsson 10th

R adjusted for critical damping.

$V_C(0) = 15V \quad i_L(0) = 6mA$



a) Find R

$$\alpha = \omega_0 \Rightarrow \frac{R}{2L} = \sqrt{\frac{1}{LC}} \quad \Rightarrow R = \boxed{1250\ \Omega}$$

b) Find $i(0^+)$ + $\frac{di}{dt}$ at 0^+

$$i(0^+) = i(0^-) = \boxed{6mA}$$

$@t = 0^+ : \quad V_C + V_r + V_L = 0$

$$-15 + i(0)(1250) + V_L = 0 \quad \Rightarrow V_L(0^+) = 7.5V$$

$$L\frac{di_L}{dt} = V_L = 7.5 \Rightarrow \frac{di_L}{dt}\bigg|_{t=0^+} = \frac{V_L}{L} = \boxed{60\ \frac{A}{s}}$$

c) Find $V_c(t)$ for $t \geq 0$

Critically damped so $V_c(t) = D_1 t e^{-5000t} + D_2 e^{-5000t}$

where $S_{1,2} = -\alpha = -\frac{R}{2L} = -5000$

$@t=0: V_c(t) = 15 = D_2 \Rightarrow V_c(t) = D_1 t e^{-5000t} + 15e^{-5000t}$

$$i_c = C\frac{dV_c}{dt} = C\left[D_1 t(-5000 e^{-5000t}) + e^{-5000t}(D_1) + 15(-5000)e^{-5000t}\right]\bigg|_{t=0}$$

$$i_c(0) = -i_L(0) = -6mA = C\left[D_1 + 15(-5000)\right] \Rightarrow D_1 = 56,250 \ V/s$$

$$\boxed{V_C = 56,250te^{-5000t} + 15e^{-5000t}\ V \qquad t \geq 0}$$